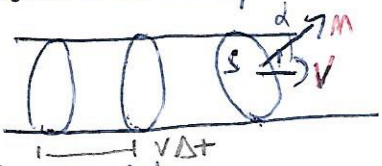
LA LEGGE DI GAUSS

massa di fluido → $\rho \cdot$ Vol partic. di Δt

Flusso di un campo vett.



$$\Delta m = \rho \overbrace{v S \Delta t \cos\alpha}^{V}$$

$$\Delta m = \rho \Delta t\, v (n S) = \rho \Delta t\, v \cdot S$$

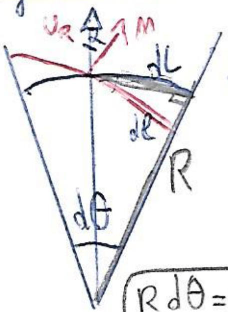
$$\boxed{d\Phi} = A \cdot n dS \equiv \boxed{A \cdot dS}$$

$(a\, v\Delta t)(b \cos\alpha)$

$v S \Delta t \cos\alpha$

$$\bar{q} = \frac{\Delta V}{\Delta t} = \frac{v S \Delta t \cos\alpha}{\Delta t} = v S \cos\alpha = \boxed{\vec{v} \cdot \vec{S}}$$

Angolo solido



$dL = dl \cos\alpha$

$$d\theta = \frac{dL}{R} \qquad d\theta = \frac{dl \cos\alpha}{R} = \frac{u_R \cdot (n\, dl)}{R}$$

$\boxed{R^2 \cdot d\Omega = dS}$

$\boxed{R\, d\theta = dL}$

$$d\Omega = \frac{dS_{sfera}}{R^2} \qquad \left(\Omega_{TOT} = \frac{1}{R^2}\int_{TOT} dS = \frac{4\pi R^2}{R^2} = 4\pi\right)$$

↳ Angolo solido tot.

$$d\Omega = \frac{dS_{sfera}}{R^2} = \frac{dS \cos\alpha}{R^2} = \frac{u_R \cdot (n\, dS)}{R^2} \equiv \frac{u_R \cdot dS}{R^2}$$

perché?

Flusso del campo elettrostatico

$$\boxed{d\Phi} = E \cdot n dS \equiv \boxed{E \cdot dS}$$

Sup. finita $\quad \Phi(E) = \int E \cdot n dS$

$\vec{dS}$

$$d\Phi = \overbrace{\frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}}^{E} \cos\alpha\, dS = \frac{q}{4\pi\varepsilon_0}\underbrace{\frac{dS_{sfera}}{r^2}}_{d\Omega} = \frac{q}{4\pi\varepsilon_0} d\Omega$$

$d\Phi$ legato solo a $d\Omega$ x legame con direzione radiale del campo e $\frac{1}{r^2}$

Legge di Gauss: Il flusso del campo el. attraverso una qualunque superficie chiusa è uguale alla somma delle cariche interne alla superficie, divisa x ε_0.

$$\boxed{\Phi_{SC}(E) = \frac{1}{\varepsilon_0}\sum_i^{int} q_i}$$

$\oint$ → la sup. su cui si calcola il flusso è chiusa.

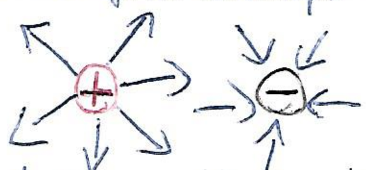
$$\Phi_{SC}(E) = \oint d\Phi = \frac{q}{4\pi\varepsilon_0}\oint d\Omega = \frac{q}{\varepsilon_0}$$

$$\boxed{\Phi_{SC}(E) = \frac{1}{\varepsilon_0}\sum_{i=1}^{n} q_i^{int}}$$

distribuzione di carica schematizzata come → continua.

$$\Phi_{SC}(E) = \frac{1}{\varepsilon_0}\int^{int} dq$$

Linee di forza del campo

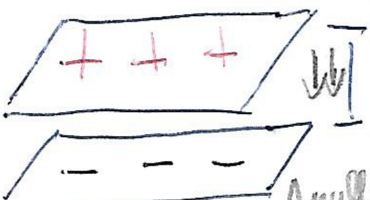


- Tg alla linea → direz. del campo.
- Verso simile.
- linee proporzionali al flusso.

Applicazioni della legge di Gauss.

$$\sigma = \frac{dq}{dS} \qquad \Phi = ES + ES = 2ES \qquad 2E\cancel{S} = \frac{\sigma \cancel{dS}}{\varepsilon_0} \Rightarrow E = \frac{\sigma}{2\varepsilon_0}$$

Annulla



Annulla.

$$\boxed{E = \frac{\sigma}{\varepsilon_0}}$$

$$2ES = \frac{q}{\varepsilon_0} \Rightarrow 2E\cancel{S} = \frac{\sigma \cancel{S}}{\varepsilon_0} \qquad E = \frac{\sigma}{2\varepsilon_0}$$

Formula locale della legge di Gauss.

teorema della divergenza

$$\oint_\Sigma A \cdot n \, dS = \int_V \operatorname{div} A \, dV \qquad \operatorname{div} A \equiv \nabla \cdot A = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\Phi_{SC}(E) = \frac{1}{\varepsilon_0} \int^{int} dq = \int_\Sigma \bar{E}\, n \, dS = \int_V \operatorname{div} E \, dV$$

deve essere nullo

$$dq = \rho \, dV \rightarrow \frac{1}{\varepsilon_0} \int^{int} dq = \frac{1}{\varepsilon_0} \int_V \rho \, dV \quad e \quad \int_V \left(\operatorname{div} E - \frac{\rho}{\varepsilon_0} \right) dV = 0 \qquad \boxed{\operatorname{div} E = \frac{\rho}{\varepsilon_0}}$$

$$\nabla \bar{E} = \frac{\rho}{\varepsilon_0}$$ legge di Gauss in forma differenziale

DERIVATE

$$f'(x_0) = \frac{f(x_0+h) - f(x_0)}{(x_0+h) - x_0} = \lim_{h\to 0} \frac{f(x_0+h) - f(x_0)}{h}$$

DERIVATA → lim del rapp. incrementale x l'incremento che tende a $\emptyset$.
≡ pendenza della retta tg in quel punto.

$D[K] = \emptyset$

$D[x] = 1$

$D[x^m] = m x^{m-1}$

$D[mx] = m\, D[x]$

$D[K \cdot f(x)] = K \cdot D[f(x)]$

Es.

– $D[2x+1]_{x=1} =$

$$f'(x_0) = \lim_{h\to 0} \frac{f(x_0+h) - f(x_0)}{h}$$

$f(x_0) = 2x_0 + 1$
$f(x_0+h) = 2(x_0+h) + 1$

$$= \lim_{h\to 0} \frac{\cancel{2x_0} + 2h \cancel{+1} \cancel{-2x_0} \cancel{-1}}{h} = \lim_{h\to 0} \frac{2\cancel{h}}{\cancel{h}} = 2$$

– $D[\sqrt{x}]_{x=1} =$

$$f'(x_0) = \lim_{h\to 0} \frac{f(x_0+h) - f(x_0)}{h}$$

$f(x_0) = \sqrt{x_0}$
$f(x_0+h) = \sqrt{x_0+h}$

$$= \lim_{h\to 0} \frac{\sqrt{x_0+h} - \sqrt{x_0}}{h} = \lim_{h\to 0} \frac{\sqrt{1+h} - 1}{h} \cdot \frac{(\sqrt{1+h}+1)}{(\sqrt{1+h}+1)} = \lim_{h\to 0} \frac{\cancel{1} + h \cancel{-1}}{h(\sqrt{1+h}+1)} = \frac{1}{1+1} = \frac{1}{2}$$

$D[\text{sen}\,x] = \cos x$

$D[\cos x] = -\text{sen}\,x$

– $D[\text{sen}\,x] =$

$$f'(x_0) = \lim_{h\to 0} \frac{f(x_0+h) - f(x_0)}{h}$$

$f(x_0) = \text{sen}\,x_0$
$f(x_0+h) = \text{sen}(x_0+h)$

$$= \lim_{h\to 0} \frac{\text{sen}(x_0+h) - \text{sen}\,x_0}{h} = \frac{\text{sen}\,x_0 \cdot \cos h + \cos x_0 \cdot \text{sen}\,h - \text{sen}\,x_0}{h} = \frac{\text{sen}\,x_0(\cos h - 1)}{h} + \frac{\cos x_0 \cdot \text{sen}\,h}{h} =$$

$$= \text{sen}\,x_0 \overset{\parallel\,\emptyset}{\left(\lim_{h\to 0} \frac{\cos h - 1}{h}\right)} + \cos x_0 \overset{\parallel\,1}{\left(\lim_{h\to 0} \frac{\text{sen}\,h}{h}\right)} = \cos x_0 \cdot 1 = \cos x_0$$

– $D[\cos x] =$

$$f'(x_0) = \lim_{h\to 0} \frac{f(x_0+h) - f(x_0)}{h}$$

$f(x_0) = \cos x_0$
$f(x_0+h) = \cos(x_0+h)$

$$= \lim_{h\to 0} \frac{\cos(x_0+h) - \cos x_0}{h} = \frac{\cos x_0 \cdot \cos h - \text{sen}\,x_0 \cdot \text{sen}\,h - \cos x_0}{h} = \frac{\cos x_0(\cos h - 1)}{h} - \frac{\text{sen}\,x_0 \cdot \text{sen}\,h}{h} =$$

$$= \cos x_0 \overset{\parallel\,\emptyset}{\left(\lim_{h\to 0} \frac{\cos h - 1}{h}\right)} - \text{sen}\,x_0 \overset{\parallel\,1}{\left(\lim_{h\to 0} \frac{\text{sen}\,h}{h}\right)} = -\text{sen}\,x_0$$

$D[\log x] = \frac{1}{x}$ $\quad$ $D[\log_a x] = \frac{1}{x} \log_a e$

$D[e^x] = e^x$ $\quad$ $D[a^x] = a^x \log a$

$D[\mathrm{Tg}\, x] = \frac{1}{\cos^2 x} = 1 + \mathrm{Tg}^2 x$

$D[\mathrm{cTg}\, x] = -\frac{1}{\mathrm{sen}^2 x}$

$D[\mathrm{arcsen}\, x] = \frac{1}{\sqrt{1-x^2}}$

$D[\mathrm{arcTg}\, x] = \frac{1}{1+x^2}$

Es.

- $y = \mathrm{sen}(x^2) \rightarrow y' = \cos(x^2) \cdot 2x$
- $y = 3 \log x \rightarrow y' = \frac{3}{x}$
- $y = \log (2x)^3 \rightarrow y' = \frac{1}{(2x)^3} \cdot 3(2x)^2 \cdot 2$
- $y = \mathrm{sen}(\log(x^3)) \rightarrow y' = \cos(\log(x^3)) \cdot \frac{1}{x^3} \cdot 3x^2$

$$\boxed{\begin{array}{l} y = f(x) \pm g(x) \Rightarrow y' = f'(x) \pm g'(x) \\ y = f(x) \cdot g(x) \Rightarrow y' = f'(x) \cdot g(x) + f(x) \cdot g'(x) \\ y = \dfrac{f(x)}{g(x)} \Longrightarrow y' = \dfrac{f'(x) \cdot g(x) - f(x) \cdot g'(x)}{[g(x)]^2} \end{array}}$$

Es.

- $y = e^{\mathrm{sen}x} + 7 - (x^{10}+x)^{32} \Rightarrow y' = e^{\mathrm{sen}x} \cdot \cos x + 0 - 32 (x^{10}+x)^{31} \cdot (10x^9+1)$
- $y = x^2 \cdot \mathrm{tg}\, x \Rightarrow y' = 2x \cdot \mathrm{Tg}\, x + x^2 \cdot \frac{1}{\cos^2 x}$
- $y = \frac{\mathrm{sen}\, x}{\cos x} \Rightarrow y' = \frac{\cos x \cdot \cos x - \mathrm{sex}(-\mathrm{sen}\, x)}{\cos^2 x} = \frac{\cos^2 x + \mathrm{sen}^2 x}{\cos^2 x} = \frac{1}{\cos^2 x}$

$D[\sqrt{x}] = \frac{1}{2\sqrt{x}}$ $\longrightarrow$ $D[x^{1/2}] = \frac{1}{2} x^{\frac{1}{2}-1} = \frac{1}{2} x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$

$D[\sqrt[n]{x}] = \frac{1}{n \sqrt[n]{x^{n-1}}}$

$D[\sqrt[n]{x^m}] = \frac{m}{n \sqrt[n]{x^{n-m}}}$

Es.

- $y = \log_2(\sqrt{x}+1) \longrightarrow y' = \frac{1}{\sqrt{x}+1} \log_2 e \cdot \left(\frac{1}{2\sqrt{x}}+0\right) = \frac{\log_2 e}{2x+2\sqrt{x}}$
- $y = \log_2(\sqrt{x+1}) \longrightarrow y' = \frac{1}{\sqrt{x+1}} \log_2 e \cdot \frac{1}{2\sqrt{x+1}} \cdot (1+0) = \frac{\log_2 e}{2(x+1)}$
- $y = \frac{\operatorname{sen} x}{1+\cos x} \longrightarrow y' = \frac{\cos x(1+\cos x) - \operatorname{sen} x(-\operatorname{sen} x)}{(1+\cos x)^2} = \frac{\cos x + \cos^2 x + \operatorname{sen}^2 x}{(1+\cos x)^2} =$

$$= \frac{\cancel{\cos x + 1}}{(1+\cos x)^{\cancel{2}}} = \frac{1}{1+\cos x}$$

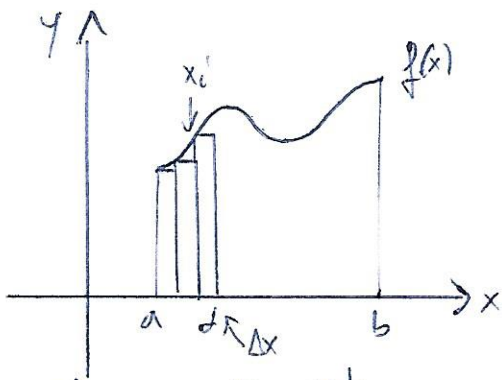
- $y = \sqrt{\operatorname{tg} x} \longrightarrow y' = \frac{1}{2\sqrt{\operatorname{tg} x}} \cdot \frac{1}{\cos^2 x}$
- $y = \operatorname{tg}(4x^2+5) \longrightarrow y' = \frac{1}{\cos^2(4x^2+5)} \cdot 8x$
- $y = \ln\left(\operatorname{tg}\frac{x}{2}\right) \longrightarrow y' = \frac{1}{\operatorname{tg}\frac{x}{2}} \cdot \frac{1}{\cos^2\left(\frac{x}{2}\right)} \cdot \frac{1}{2} = \frac{\cancel{\cos\frac{x}{2}}}{\operatorname{sen}\frac{x}{2}} \cdot \frac{1}{\cos^{\cancel{2}}\frac{x}{2}} =$

$$= \frac{1}{2\operatorname{sen}\frac{x}{2}\cos\frac{x}{2}} = \frac{1}{\operatorname{sen} x} \qquad \left(\begin{array}{l}\operatorname{sen} 2\alpha = 2\operatorname{sen}\alpha\cdot\cos\alpha \\ \frac{x}{2} = \alpha\end{array}\right)$$

- $y = \sqrt[3]{\operatorname{tg}^2 x} \longrightarrow y' = \frac{2}{3\sqrt[3]{(\operatorname{tg} x)^{3-2}}} \cdot \frac{1}{\cos^2 x} = \frac{2}{3\sqrt[3]{\operatorname{tg} x}\cdot\cos^2 x}$

$\left[\frac{\infty}{\infty}\right]$ $\left[\frac{0}{0}\right]$ $[0\cdot\infty]$ $[\infty^0]$ $[0^0]$ $[1^\infty]$ $[+\infty-\infty]$ 7 forme indeterminate.

INTEGRALI



$$\sum_{i=1}^{n} \delta\cdot f(x_i) \rightarrow \text{Area approssimata}$$

$$\lim_{\substack{n\to\infty \\ (\delta\to 0)}} \sum_{i=1}^{n} \delta\cdot f(x_i) = \int f(x)\cdot\Delta x = \boxed{\int_a^b f(x)\cdot dx}$$

$f(x) \rightarrow$ continua. $F(x)$ primitiva di $f(x)$ se $F'(x) = f(x)$

$$\int_a^b f(x)\,dx = \left[F(x)\right]_a^b = F(b) - F(a)$$

Primitive $\rightarrow$ difficili da calcolare $\rightarrow$ sono una famiglia che prende il nome di integrale indefinito (in quanto possono differenziarsi x una cost.)

$$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1} + c$$

$$\int_0^{2\pi} \operatorname{sen} x\, dx = \left[-\cos x\right]_0^{2\pi} = -\cos 2\pi - (-\cos\phi) = -1+1 = \phi$$

PROPRIETÀ DELL' INTEGRALE INDEFINITO

I) $\int [f(x) \pm g(x)]\,dx = \int f(x)\,dx \pm \int g(x)\,dx$

II) $\int k\,f(x)\,dx = k\int f(x)\,dx$

Es.

- $\int \left[\frac{3}{x^4} - 5\sqrt[3]{x} - \frac{2}{\sqrt[5]{x^2}}\right]dx = 3\int \frac{1}{x^4}\,dx - 5\int \sqrt[3]{x}\,dx - 2\int \frac{1}{\sqrt[5]{x^2}}\,dx =$

$= 3\int x^{-4}\,dx - 5\int x^{1/3}\,dx - \int x^{-2/5}\,dx = 3\,\frac{x^{-4+1}}{-4+1} - 5\,\frac{x^{1/3+1}}{\frac{1}{3}+1} - 2\,\frac{x^{-2/5+1}}{-\frac{2}{5}+1} =$

$= -\frac{1}{x^3} - \frac{15}{4}\sqrt[3]{x^4} - \frac{10}{3}\sqrt[5]{x^3} + c$

$\int [f(x)]^{\alpha}\cdot f'(x)\,dx = \frac{[f(x)]^{\alpha+1}}{\alpha+1} + c$

Es.

- $\int \underbrace{(2x-1)}_{f}{}^{\overset{\alpha}{3}}\cdot \underbrace{2}_{f'}\,dx = \frac{(2x-1)^4}{4} + c$
- $\int \frac{x}{\sqrt{x^2+12}}\,dx = \int (x^2+12)^{-1/2}\cdot x\,dx = \frac{2}{2}\int (x^2+12)^{-1/2}\cdot x\,dx = \frac{1}{2}\int (x^2+12)^{-1/2}\cdot 2x\,dx =$

$= \frac{1}{2}\cdot\frac{(x^2+12)^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} + c = \sqrt{(x^2+12)} + c$

$\int \frac{1}{x}\,dx = \ln|x| + c$

$\int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)| + c$

Es.

- $\int \frac{x^2}{x^3+2}\,dx = \frac{3}{3}\int \frac{x^2}{x^3+2}\,dx = \frac{1}{3}\int \frac{3x^2}{x^3+2}\,dx = \frac{1}{3}\ln|x^3+2| + c$

INTEGRALI IMMEDIATI

$$\int [f(x)]^{\alpha} \cdot f'(x) = \frac{[f(x)]^{\alpha+1}}{\alpha+1} + c$$

$$\int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)| + c$$

$$\int e^{f(x)} \cdot f'(x)\,dx = e^{f(x)} + c \qquad \int a^{f(x)} \cdot f'(x) = \frac{a^{f(x)}}{\ln a} + c$$

$$\int \cos f(x) \cdot f'(x)\,dx = \operatorname{sen} f(x) + c$$

$$\int \operatorname{sen} f(x) \cdot f'(x)\,dx = -\cos f(x) + c$$

$$\int \frac{1}{\cos^2 f(x)} \cdot f'(x) = \operatorname{tg} f(x) + c$$

$$\int \frac{1}{\operatorname{sen}^2 f(x)} \cdot f'(x) = -\operatorname{ctg} f(x) + c$$

$$\int \frac{1}{1+[f(x)]^2} \cdot f'(x) = \operatorname{arctg} f(x) + c$$

$$\int \frac{1}{\sqrt{1-[f(x)]^2}} \cdot f'(x) = \operatorname{arcsen} f(x) + c$$